Commodity Connectedness

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Financial and Macroeconomic Connectedness

- Portfolio concentration risk
- Credit risk

- Counterparty and gridlock risk
- Systemic risk (including MES, CoVaR, system-wide, ...)

- Business cycle risk

- This paper: Commodities
  – Crucial part of the global economy
  – Partly financial, partly real
  – Key inputs and key outputs
  – Static and dynamic aspects
  – Real-time monitoring of commodity market volatility

  "Real-time monitoring for real-time policy"

Covariance

- So pairwise...
- So linear...
- So Gaussian...

A Very General Environment

$$x_t = B(L)\,\varepsilon_t$$

$$\varepsilon_t \sim (0, \Sigma)$$

$$\boxed{C(x, B(L), \Sigma)}$$

A Natural Economic Connectedness Question:

What fraction of the H-step-ahead prediction-error variance of variable i is due to shocks in variable j, $j \neq i$?

Non-own elements of the variance decomposition: $d_{ij}^H, \; j \neq i$

$$\boxed{C(x, H, B(L), \Sigma)}$$

Variance Decompositions for Connectedness

N-Variable Connectedness Table

	x_1	x_2	...	x_N	From Others to i
x_1	d^H_{11}	d^H_{12}	$\cdots$	d^H_{1N}	$\Sigma^N_{j=1} d^H_{1j}, j \neq 1$
x_2	d^H_{21}	d^H_{22}	$\cdots$	d^H_{2N}	$\Sigma^N_{j=1} d^H_{2j}, j \neq 2$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d^H_{N1}	d^H_{N2}	$\cdots$	d^H_{NN}	$\Sigma^N_{j=1} d^H_{Nj}, j \neq N$
To Others From j	$\Sigma^N_{i=1} d^H_{i1}$ $i \neq 1$	$\Sigma^N_{i=1} d^H_{i2}$ $i \neq 2$	$\cdots$	$\Sigma^N_{i=1} d^H_{iN}$ $i \neq N$	$\Sigma^N_{i,j=1} d^H_{ij}$ $i \neq j$

Upper-left block is variance decomposition matrix, D

Connectedness involves the **non-diagonal** elements of D

Connectedness Measures

- Pairwise Directional: $C^H_{i \leftarrow j} = d^H_{ij}$ (“i’s imports from j”)
- Net: $C^H_{ij} = C^H_{j \leftarrow i} - C^H_{i \leftarrow j}$ (“ij bilateral trade balance”)

- Total Directional:
  - From others to i: $C^H_{i \leftarrow \bullet} = \sum_{\substack{j=1 \\ j \neq i}}^{N} d^H_{ij}$ (“i’s total imports”)
  - To others from j: $C^H_{\bullet \leftarrow j} = \sum_{\substack{i=1 \\ i \neq j}}^{N} d^H_{ij}$ (“j’s total exports”)
- Net: $C^H_{i} = C^H_{\bullet \leftarrow i} - C^H_{i \leftarrow \bullet}$ (“i’s multilateral trade balance”)

- Total System-Wide: $C^H = \frac{1}{N} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} d^H_{ij}$ (“total world exports”)

Background

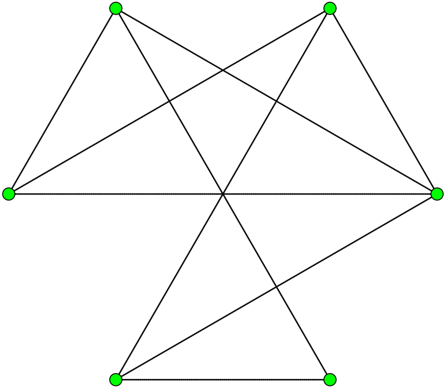
Recent paper:

Diebold, F.X. and Yilmaz, K. (2014), "On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms," *Journal of Econometrics*, 182, 119-134.

Recent book:

Diebold, F.X. and Yilmaz, K. (2015), *Financial and Macroeconomic Connectedness: A Network Approach to Measurement and Monitoring*, Oxford University Press. With K. Yilmaz.

Network Representation: Graph and Matrix



$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

Symmetric adjacency matrix A
$A_{ij} = 1$ if nodes i, j linked
$A_{ij} = 0$ otherwise

Network Connectedness: The Degree Distribution

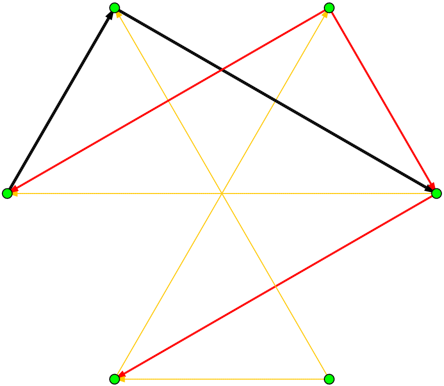
Degree of node i, d_i:

$$d_i = \sum_{j=1}^{N} A_{ij}$$

Discrete *degree distribution* on 0, ..., $N - 1$

Mean degree, $E(d)$, is the key connectedness measure

Network Representation II (Weighted, Directed)



$$A = \begin{pmatrix} 0 & .5 & .7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & .3 & 0 \\ 0 & 0 & 0 & .7 & 0 & .3 \\ .3 & .5 & 0 & 0 & 0 & 0 \\ .5 & 0 & 0 & 0 & 0 & .3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

"to i, from j"

Network Connectedness II: The Degree Distribution(s)

$A_{ij} \in [0, 1]$ depending on connection strength

Two degrees:

$$d_i^{from} = \sum_{j=1}^{N} A_{ij}$$

$$d_j^{to} = \sum_{i=1}^{N} A_{ij}$$

"from-degree" and "to-degree" distributions on $[0, N-1]$

Mean degree remains the key connectedness measure

Variance Decompositions as Weighted, Directed Networks

Variance Decomposition / Connectedness Table

	x_1	x_2	...	x_N	From Others
x_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H	$\sum_{j\neq 1} d_{1j}^H$
x_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H	$\sum_{j\neq 2} d_{2j}^H$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H	$\sum_{j\neq N} d_{Nj}^H$
To Others	$\sum_{i\neq 1} d_{i1}^H$	$\sum_{i\neq 2} d_{i2}^H$	$\cdots$	$\sum_{i\neq N} d_{iN}^H$	$\sum_{i\neq j} d_{ij}^H$

Total directional "from", $C_{i\leftarrow\bullet}^H = \sum_{\substack{j=1\\ j\neq i}}^N d_{ij}^H$: "from-degrees"

Total directional "to", $C_{\bullet\leftarrow j}^H = \sum_{\substack{i=1\\ i\neq j}}^N d_{ij}^H$: "to-degrees"

Total system-wide, $C^H = \frac{1}{N} \sum_{\substack{i,j=1\\ i\neq j}}^N d_{ij}^H$: mean degree

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Estimating Connectedness

Thus far we've worked under correct specification, in population:

$$C(x, H, B(L), \Sigma)$$

Now we want:

$$\widehat{C}\left(x, H, B(L), \Sigma, M(L;\hat{\theta})\right),$$

and similarly for other variants of connectedness

Many Interesting Issues / Choices

- x objects: Returns? **Return volatilities**?
- x universe: How many and which ones? (**Major commodity sub-indexes**)
- x frequency: **Daily?** Monthly? Quarterly?

- Specification: Approximating model M: **VAR**? DSGE?
- Estimation: Classical? Bayesian? **Hybrid**?
  - Selection: Information criteria? Stepwise? **Lasso**?
  - Shrinkage: BVAR? Ridge? **Lasso**?
- Identification (of variance decompositions):
  - Assumptions: Cholesky? **Generalized**? SVAR? DSGE?
  - Horizon H: **10-day**? Others?
- Understanding: **Network summarization and visualization**
- Estimation: Static vs. dynamic

Selection and Shrinkage via Penalized Estimation of High-Dimensional Approximating Models

$$\hat{\beta} = argmin_{\beta} \sum_{t=1}^{T} \left(y_t - \sum_i \beta_i x_{it} \right)^2 \quad s.t. \quad \sum_{i=1}^{K} |\beta_i|^q \leq c$$

$$\hat{\beta} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_i \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta_i|^q \right)$$

Concave penalty functions non-differentiable at the origin produce selection. Smooth convex penalties produce shrinkage. $q \to 0$ produces selection, $q = 2$ produces ridge, $q = 1$ produces lasso.

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Lasso

$$\hat{\beta}_{Lasso} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta_i| \right)$$

$$\hat{\beta}_{AEnet} = argmin_{\beta} \left(\sum_{t=1}^{T} \left(y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} w_i \left(\alpha|\beta_i| + (1-\alpha)\beta_i^2 \right) \right)$$

where $w_i = 1/|\hat{\beta}_i|^{\nu}$, $\hat{\beta}_i$ is OLS or ridge, and $\nu > 0$.

Choices: $\alpha = 0.5$; w_i from OLS regression; $\nu = 1$; 10-fold cross validation for λ, separately for each equation.

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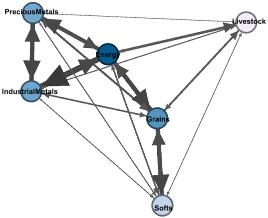
Network Visualization via "Spring Graphs"



- Node shading: Total directional connectedness "to others"
- Node location: Average pairwise directional connectedness (Equilibrium of repelling and attracting forces, where (1) nodes repel each other, but (2) edges attract the nodes they connect according to average pairwise directional connectedness "to" and "from.")
- Link thickness: Average pairwise directional connectedness
- Link arrow sizes: Pairwise directional "to" and "from"

Commodity Contracts

Commodity	Designated Contract	Exchange
Natural Gas	Henry Hub Natural Gas	NYMEX
WTI Crude Oil	Light, Sweet Crude Oil	NYMEX
Unleaded Gasoline	RBOB	NYMEX
ULS Diesel (Heating Oil)	ULS Diesel	NYMEX
Live Cattle	Live Cattle	CME
Lean Hogs	Lean Hogs	CME
Wheat	Soft Wheat	CBOT
Corn	Corn	CBOT
Soybeans	Soybeans	CBOT
Soybean Oil	Soybean Oil	CBOT
Aluminum	High Grade Primary Aluminum	LME
Copper	Copper	COMEX
Zinc	Special High Grade Zinc	LME
Nickel	Primary Nickel	LME
Gold	Gold	COMEX
Silver	Silver	COMEX
Sugar	World Sugar No. 11	NYBOT
Cotton	Cotton	NYBOT
Coffee	Coffee ‘C’	NYBOT

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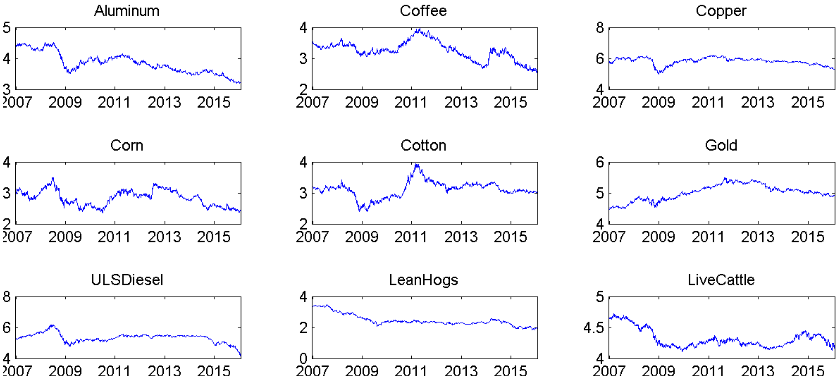
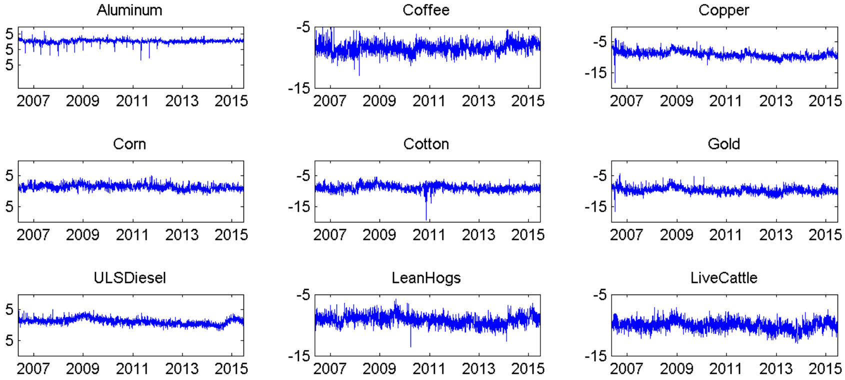
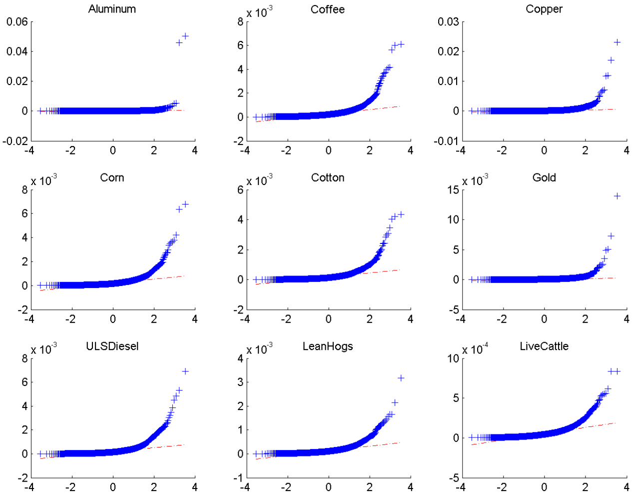
Time Series Plots of Log Commodity Sub-Indices



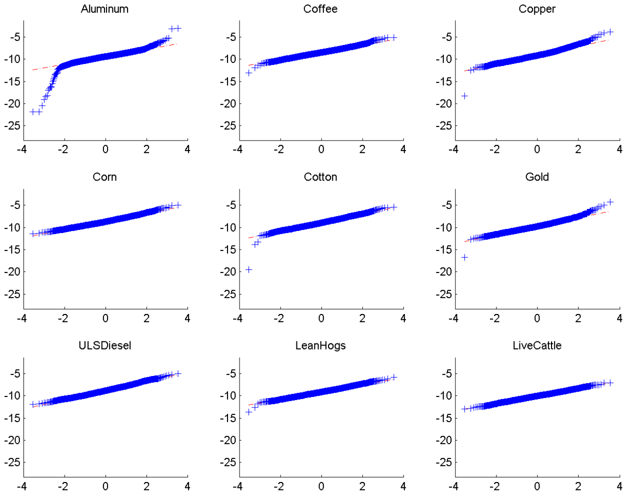
Time Series Plots of Log Realized Volatilities



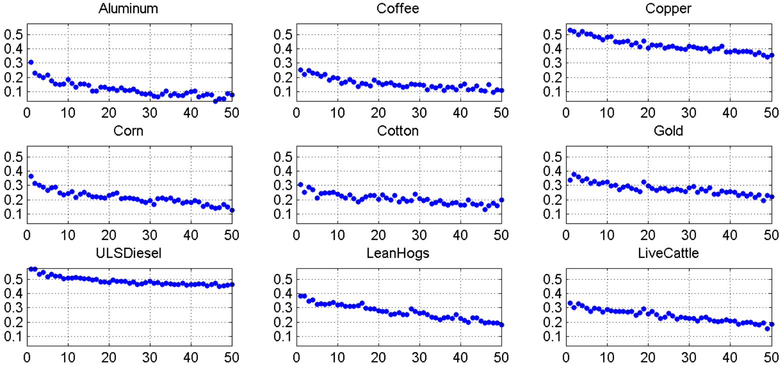
Gaussian Q-Q Plots for Realized Volatilities



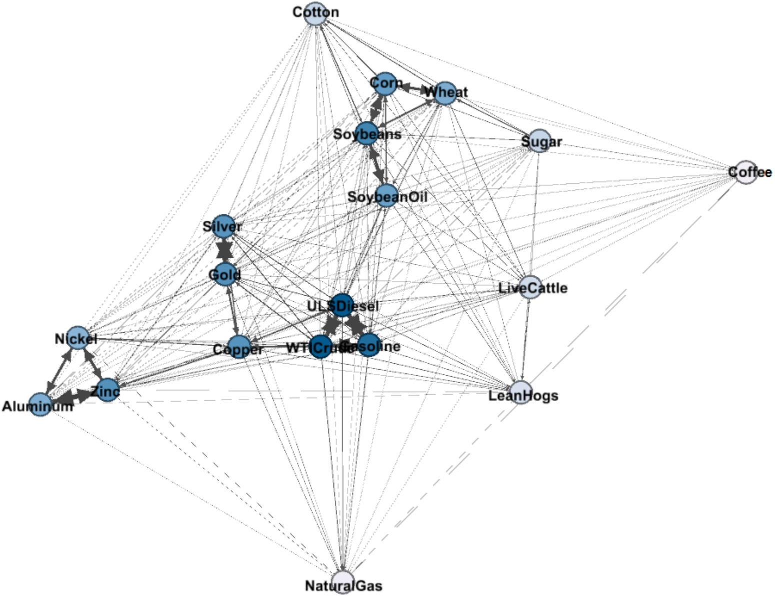
Gaussian Q-Q Plots for Log Realized Volatilities



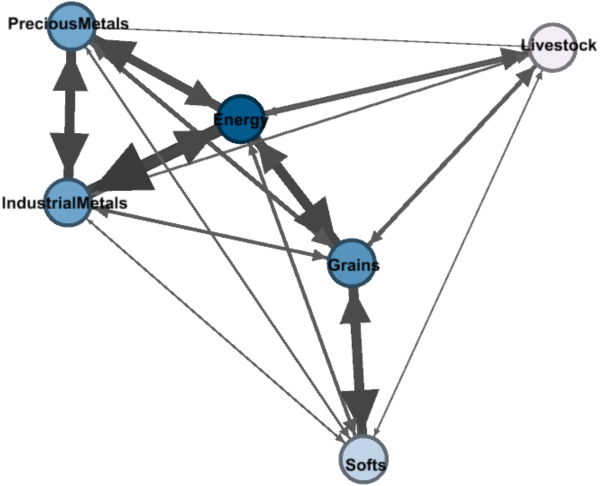
Autocorrelation Functions of Log Realized Volatilities



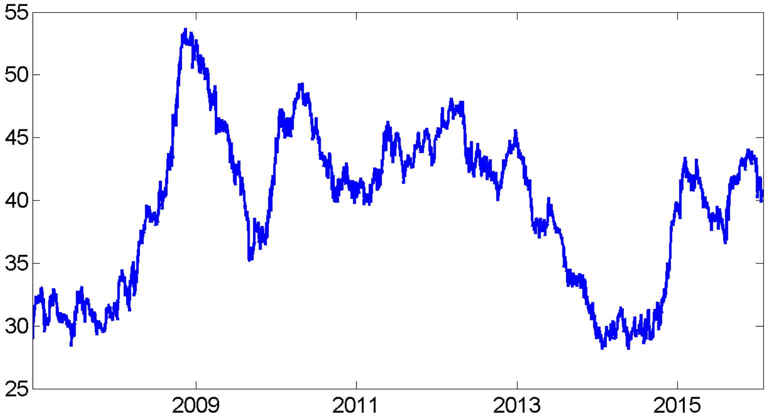
Full-Sample Spring Graph



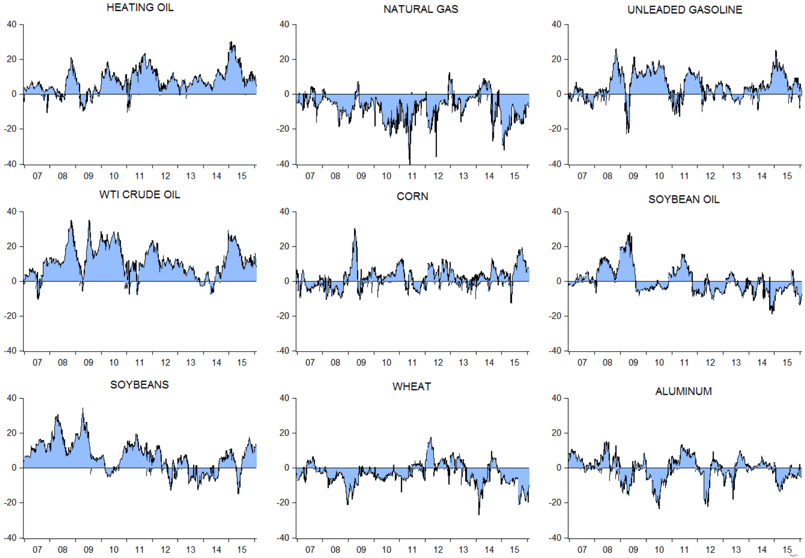
Full-Sample Spring Graph, Six-Group Aggregation



Rolling-Sample System-Wide Connectedness



Rolling-Sample Net Total Directional Connectedness



Conclusion